Adaptive Neural Control for a Class of Nonlinear Time-Varying Delay Systems With Unknown Hysteresis

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***Abstract*— This paper investigates the fusion of unknown direction hysteresis model with adaptive neural control techniques in face of time-delayed continuous time nonlinear systems without strict-feedback form. Compared with previous works on the hysteresis phenomenon, the direction of the modified Bouc–Wen hysteresis model investigated in the literature is unknown. To reduce the computation burden in adaptation mechanism, an optimized adaptation method is successfully applied to the control design. Based on the Lyapunov–Krasovskii method, two neural-network-based adaptive control algorithms are constructed to guarantee that all the system states and adaptive parameters remain bounded, and the tracking error converges to an adjustable neighborhood of the origin. In final, some numerical examples are provided to validate the effectiveness of the proposed control methods.**

***Index Terms*— Adaptive neural control, Bouc–Wen hysteresis, nonlinear control, nonstrict-feedback system, unknown direction hysteresis.**

I. Introduction

OVER THE past few decades, approximation-based adaptive backstepping control for nonlinear systems has received a great deal of attention, and some significant results are presented in [1]–[20]. Among them, [1]–[7] are for strict-feedback systems in single-input and single-output form and [8]–[11] research the multiple-input and multiple-output (MIMO) nonlinear systems, while the nonlinear systems with immeasurable states or time delays are investigated in [4], [5], and [13]–[20]. In these control schemes, fuzzy logic systems (FLSs) or neural networks (NNs) are employed to approximate the unknown nonlinear functions. Based on the backstepping or dynamic surface control (DSC) technique, adaptive fuzzy controller or adaptive neural controller are constructed to guarantee a good tracking performance. However, all the aforementioned control schemes are only suitable to the nonlinear systems in strict-feedback form, which is in fact a special case of the nonstrict-feedback form proposed in [21] recently.

Different from the conventional strict-feedback systems, the semistrict-feedback systems, or the affine pure-feedback systems, the unknown nonlinear functions in the nonstrict-feedback system contain whole state variable, i.e., they cannot be directly compensated by the approximators with current states. In addition, the virtual control signals require to be the functions of current states to guarantee their existence. To deal with these difficulties, Chen *et al.* [21] have proposed a variable separation technique. Based on the Lyapunov synthesis and the FLSs technique, an adaptive control scheme was constructed in [21] to guarantee the stability of nonstrict-feedback control system. Although much progress has been made in adaptive control field in [1]–[19] and [21], some challenging problems still remain. In practical application, the actuator of control system is often subjected to the nonsmooth characteristics, such as the hysteresis nonlinearity in [29], [30], [34]–[39], [41], [45], [57], and [74], the saturation nonlinearity in [22]–[28], [31], and [33], the dead-zone nonlinearity in [20], [43], and [44], and the backlash nonlinearity in [40]. Especially, the hysteresis phenomenon in the physical systems and devices severely limits the tracking performance and easily causes the instability [57]. Therefore, control of nonlinear systems preceded by the nonsmooth hysteresis characteristic has become one of the active topics.

To address the control problem of nonsmooth hysteresis nonlinearity in the actuator, the authors in [30], [34]–[39], [41], [45], [57], and [74] have made outstanding contributions in the fusion of hysteresis models with available robust control techniques. These hysteresis models can be classified into four classes, which are the backlash-like hysteresis model in [30], [34], [45], and [74], the conventional Prandtl–Ishlinskii (P–I) hysteresis model in [35] and [36], the generalized P–I hysteresis model in [37] and [42], and the modified Bouc–Wen hysteresis model in [41], respectively. In [34], the backlash-like hysteresis model was first proposed to describe a class of

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special hysteresis phenomenon in the actuator. By treating the effect of backlash-like hysteresis as a bounded disturbance, the proposed model can be conveniently fused with available robust control techniques. Furthermore, Su *et al.* [35] have fused a more general conventional P–I hysteresis model with adaptive variable structure control approach. Based on the P–I model, an adaptive control algorithm was proposed in [35] to guarantee a good tracking performance. To enlarge the class of applicable systems, Ren *et al.* [36] considered a class of MIMO nonlinear systems with the conventional P–I hysteresis input and time-varying delays. By using the NNs technique to approximate the unknown nonlinear functions, the restriction of the linearly parameterized unknown uncertainties can be removed. The effect of time-varying delays can be canceled by designing an appropriate Lyapunov–Krasovskii function. Therefore, an approximation-based adaptive control scheme was developed in [36] to guarantee the semiglobally uniformly ultimately bounded of control system. To accommodate more general classes of hysteresis shapes, a generalized P–I hysteresis model was proposed in [42]. For the conventional P–I hysteresis model, the different hysteresis shapes are described by adjusting the density function only. For the generalized case, both the density function and the input function can be utilized to formulate the different hysteresis shapes, i.e., compared with the conventional case, the generalized P–I hysteresis model can capture the hysteresis phenomenon more accurately. Ren *et al.* [37] considered a class of nonaffine pure-feedback nonlinear systems with the generalized P–I hysteresis input. To deal with the nonaffine problem in the presence of nonsmooth characteristic of hysteresis, the mean-value theorem was introduced. By using the backstepping and NNs techniques, the control approach in [37] can guarantee the uniformly ultimately bounded of control system, and the tracking error converges to a small neighborhood of the origin. To relax the restrictions about the density function in P–I hysteresis model, a modified Bouc–Wen hysteresis model was first proposed in [41]. By constructing a perfect inverse to compensate the effect of hysteresis nonlinearity, an adaptive control approach was provided in [41] to guarantee a good tracking performance. However, the directions of the aforementioned four classes of hysteresis models require to be known. If the direction of hysteresis phenomenon in the actuator is unknown, the control approaches mentioned above cannot be directly applied.

Motivated by the aforementioned researches, this paper focuses on the problem of adaptive neural control for a class of continuous time (CT) nonstrict-feedback systems with unknown direction modified Bouc–Wen hysteresis input and time-varying delays. The main contributions in this literature can be summarized as follows.

1) The hysteresis models studied in [29], [30], [34]–[39], [41], [45], [57], and [74] are known direction hysteresis, i.e., though the hysteresis parameter (it refers to the coefficient or the function governing the hysteresis direction) may be unknown, its sign must be known as prior knowledge. If the hysteresis direction is unknown, the aforementioned control schemes cannot be directly applied. In this paper, we investigate the fusion of the unknown direction hysteresis with adaptive neural control approaches.
2) Compared with the unknown nonlinear functions of current states in conventional strict-feedback systems, the unknown functions in the nonstrict-feedback system contain whole state variables, i.e., more NN nodes should be utilized to obtain a good approximated performance. If we still employ the conventional adaptation mechanism, it will lead to unacceptable large learning time as the number of NN nodes increases. To solve these difficulties in face of the control system without strict-feedback form, an optimized method is included in the adaptation mechanism in this literature. By employing the optimized adaptation method, the number of adaptive parameters does not increase as the NN nodes increase, and the less knowledge about the centers and widths of basis functions are needed for the controller design.
3) Compared with the nonlinear systems in Brunovsky form or nonaffine pure-feedback form studied in [34]–[41], the control system investigated in this paper is the CT nonstrict-feedback form, which is in fact a more general case [21]. Therefore, the investigation in this paper enlarges the class of applicable systems with hysteresis phenomenon.

II. Problem Formulation and Some Preliminaries

A. Nonlinear Control Problem

Consider the following nonstrict-feedback system with unknown hysteresis input and time-varying delays:

$$\begin{aligned} \dot{x} &= Ax + f(x) + h(x(\tau(t))) + Bu + \Delta(t,x) \\ u &= H(\upsilon), \quad y = C^T x \end{aligned} \tag{1}$$

where

$$\begin{aligned} A &= \begin{bmatrix} 0 & I_{n-1} \\ 0 & 0 \end{bmatrix}, \quad B = [0,\ldots,0,1]^T \in R^n \\ C &= [1,0,\ldots,0]^T \in R^n \\ f(x) &= [\ldots, f_i(x), \ldots, f_n(x)]^T, \quad i = 1,\ldots,n-1 \\ h(x(\tau(t))) &= [\ldots, h_i(x(t-\tau_i(t))), \ldots, h_n(x(t-\tau_n(t)))]^T \\ \Delta(t,x) &= [\ldots, \omega_i(t,x), \ldots, \omega_n(t,x)]^T \end{aligned} \tag{2}$$

where $x = [x_1, x_2, \ldots, x_n]^T \in R^n$ are the whole state variables of control system. $f(x) \in R^n$, $\upsilon \in R$, and $y(t) \in R$ denote the unknown smooth function of whole states, system input, and system output, respectively. $\Delta(t,x) \in R^n$ are the external disturbances and $h(x(\tau(t))) \in R^n$ are the unknown smooth functions of time-varying time-delayed states. I_{n-1} is an identity matrix. Here, the control signal is subjected to the unknown hysteresis nonlinearity $H(\upsilon) : R \to R$ given in the later.

Remark 1: Different from the conventional strict-feedback nonlinear systems, the unknown functions $f_i(x)$, $h_i(x)$, and $\omega_i(t,x)$ in the differentiation (1) are evidently the functions of whole state variables. Hence, these terms cannot be dealt with as done in most of the previous works, where they are approximated by the NNs of current states. To solve the difficulty, Chen *et al.* [21] have proposed a variable separation technique. However, the fusion of unknown direction hysteresis with these adaptive control techniques is still a challenging problem, which directly motivates the investigation of this paper.

Remark 2: The hysteresis phenomenon has been investigated in many previous works, such as the backlash-like hysteresis model in [30], [34], [45], and [74], the conventional P–I hysteresis model in [35] and [36], the generalized P–I hysteresis model in [37] and [42], and the modified Bouc–Wen hysteresis model in [41]. However, the directions of these hysteresis models require to be known. If the practical hysteresis direction is unknown, the aforementioned control schemes cannot be directly applied. To the best of our knowledge, it is the first time, in the literature, that the fusion of unknown direction hysteresis with adaptive neural control techniques is investigated.

The objective of this paper is to design a robust adaptive neural controller for system (1) to guarantee two points.

1) All the system states and the adaptive parameters are bounded.
2) The tracking error $z_1(t) = y(t) - y_r(t)$ converges to an adjustable neighborhood of the origin, where $y(t)$ is the output of control system and $y_r(t)$ is the reference signal. Specifically, for any $\varepsilon > 0$, there exists a time value $T(\varepsilon) > 0$ such that $z_1^2(t) \leq \varepsilon, \forall t > T$.

Assumption 1: For $1 \leq i \leq n$, there exists an unknown positive function $\varpi_i(x)$ such that

$$|\omega_i(t, x)| \leq \varpi_i(x). \tag{3}$$

Remark 3: Similar external disturbances $\varpi_i(t, x)$ were considered in [69], where they should satisfy the restriction $|w_i(t, x)| \leq \varpi_i(\bar{x}_i)$, $\bar{x}_i = [x_1, \ldots, x_i]^T \in R^i$. From this point of view, Assumption 1 is in fact a relaxed version of the restriction in [69]. Therefore, the external disturbances studied in this paper are more general.

Assumption 2: For $1 \leq i \leq n$, consider $\tau_i(t)$ is the time-varying time delay. It requires to satisfy the following inequalities:

$$0 < \tau_i(t) \leq \tau_{\max} \quad \dot{\tau}_i \leq \bar{\tau}_{\max} < 1 \tag{4}$$

where $\tau_{\max}$ and $\bar{\tau}_{\max}$ are known constants.

B. Hysteresis Nonlinearity

A modified Bouc–Wen hysteresis nonlinearity can be formulated as follows [41]:

$$H(\upsilon) = \mu_1 \upsilon + \mu_2 \zeta \tag{5}$$

where μ_1 and μ_2 are the constants with the same signs, i.e., $\text{sign}(\mu_1) = \text{sign}(\mu_2)$. ζ can be specified as

$$\dot{\zeta} = \dot{\upsilon} - \beta|\dot{\upsilon}||\zeta|^{n-1}\zeta - \chi\dot{\upsilon}|\zeta|^n = \dot{\upsilon} f(\zeta, \dot{\upsilon}), \quad \zeta(t_0) = 0 \tag{6}$$

where $\beta > |\chi|$, $n > 1$. The shape and amplitude of the hysteresis can be described by the parameters β while the smoothness from initial slope to asymptote's slope can be described by the parameter n. The auxiliary variable ζ is bounded by the constant $\bar{\zeta}$, i.e., $|\zeta| \leq \bar{\zeta} = \sqrt[n]{1/(\beta + \chi)}$. The function $f(\zeta, \dot{\upsilon})$ can be formulated as follows:

$$f(\zeta, \dot{\upsilon}) = 1 - \text{sign}(\dot{\upsilon})\beta|\zeta|^{n-1}\zeta - \chi|\zeta|^n. \tag{7}$$

TABLE I

SIMULATION PARAMETERS

hysteresis parameters	μ_1	μ_2	β	χ	n	$\zeta(t_0)$
positive direction	3	5	1.5	0.5	2	0
negative direction	-3	-5	1.5	0.5	2	0

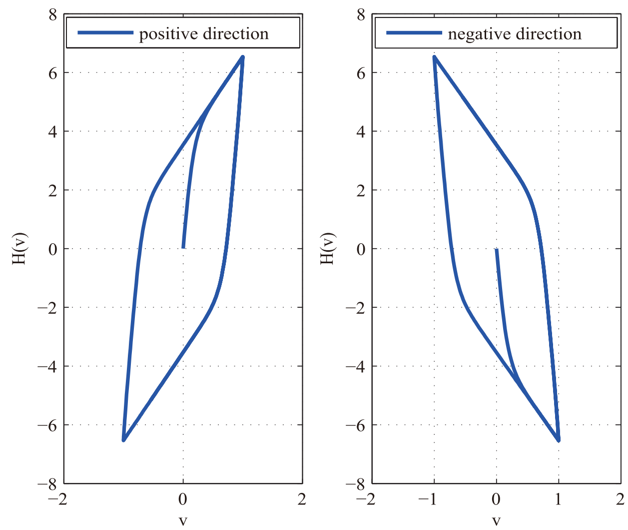

Fig. 1. Trajectories of positive direction hysteresis and negative direction hysteresis.

Remark 4: It should be noted that the sign of μ_1 governs the direction of hysteresis. The issue can be clearly illustrated by the following numerical example. The simulation parameters are given in Table I. By selecting the input signal $\upsilon(t)$ of hysteresis actuator as $\upsilon(t) = \sin(2t)$, the graphical result is shown in Fig. 1.

C. Neural Networks

In most of the robust adaptive neural control schemes, the radial basis function neural networks (RBFNNs) are utilized to deal with unknown nonlinear terms as the approximation property. The RBFNNs have the form $\theta^T S(Z)$, where $\theta = [\theta_1, \ldots, \theta_N]^T \in R^N$ is the weight vector with the integer N, $S(Z) = [S_1(Z), \ldots, S_N(Z)]^T \in R^N$ is the vector of known basis function, and $Z = [Z_1, \ldots, Z_q]^T \in R^q$ is the input vector of approximator, see in Fig. 2. Noting that the Gaussian RBFNNs have the basis function $S_i(Z)$, which uses the structure of Gaussian function and it can be formulated as follows:

$$S_i(Z) = \exp \frac{-(Z - \mu_i)^T (Z - \mu_i)}{\eta_i^2}, \quad i = 1, 2, \ldots, N \tag{8}$$

where $\mu_i = [\mu_{i1}, \ldots, \mu_{iq}]^T$ is the center of the receptive field and η_i is the width of the Gaussian function. It was proved that RBFNNs can approximate any nonlinear continuous function

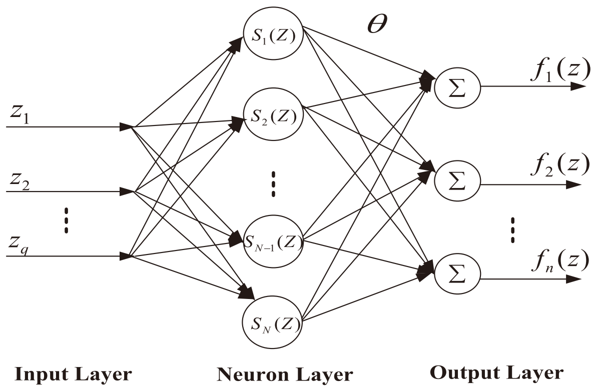

Fig. 2. Structure of RBFNN.

$f_j(Z)(j = 1, \ldots, n)$ over a compact set $Z \in \Omega$ with any accuracy. Therefore, the following lemma holds.

Lemma 1: For any $\epsilon > 0$, there exists an NN $\theta^T S(Z)$ such that

$$\sup_{Z\in\Omega} |f_j(Z) - \theta^T S(Z)| \le \epsilon \tag{9}$$

where Ω is a compact set of Z. The universal approximation property can be well utilized to deal with the unknown nonlinear terms.

D. Nussbaum Function

A function $N(\vartheta)$ is the Nussbaum function if it satisfies

$$\lim_{k\to\pm\infty} \sup \frac{1}{k}\int_0^k N(s)ds = \infty \tag{10}$$

$$\lim_{k\to\pm\infty} \inf \frac{1}{k}\int_0^k N(s)ds = -\infty. \tag{11}$$

Thus, we know that many functions satisfy the aforementioned conditions such as $\exp(\vartheta^2)\cos\vartheta$, $\vartheta^2\cos\vartheta$.

Lemma 2: Let $V(\cdot)$ and $\vartheta(\cdot)$ be smooth functions defined on $[t_0, t_f)$ with $V(t) \ge 0, \forall t \in [t_0, t_f)$, $N(\cdot)$ is an even smooth Nussbaum-type function. If the following inequality holds for $\forall t \in [t_0, t_f)$:

$$V(t) \le h_1 + \int_{t_0}^t (\xi(t)N(\vartheta) + 1)\dot{\vartheta} e^{-h_2(t-\tau)} d\tau \tag{12}$$

where h_1 and h_2 are positive scalars. $\xi(t)$ is a bounded function, which takes values at the interval $\Pi : [l^-, l^+]$ and $0 \notin \Pi$. A conclusion can be made that $V(t)$, $\vartheta(t)$, $\int_{t_0}^t (\xi N(\vartheta)+1)\dot{\vartheta} d\tau$ must be bounded on $[t_0, t_f)$. Furthermore, t_f can be extended to infinity, i.e., $t_f \to \infty$ if the solution of the resulting closed-loop system is bounded [20].

III. Adaptive Neural Control Design Based on Lyapunov Synthesis

We use the backstepping and NN techniques to construct the adaptive neural control schemes. The coordinate transformations are defined as follows:

$$z_1 = y - y_r \tag{13}$$

$$z_i = x_i - \alpha_{i-1} \quad 2 \le i \le n \tag{14}$$

where y_r is the reference signal and assuming its time derivatives up to nth order remain bounded and continuous.

Assumption 3: For $1 \le i \le n$, there exists an unknown positive function q_{ij} such that

$$|h_i(x(t))| \le \sum_{j=1}^n |z_j(t)| q_{ij}(x(t)) \tag{15}$$

where $x(t) = [x_1(t), \ldots, x_n(t)]^T$ are the whole state variables of control system. z_j is the coordinate transformation, i.e., $z_1 = y - y_r$ and $z_j = x_j - \alpha_{j-1} (2 \le j \le n)$, where α_{j-1} is the virtual controller given latter.

Remark 5: A similar assumption was made in [64]. However, the time-delayed term $h_i(\cdot)$ studied in [64] is a function of current states, i.e., $h_i(\cdot) = h_i(\bar{x}_i)$, where $\bar{x}_i = [x_1, \ldots, x_i]^T$. In this paper, we extend the time-delayed term to $h_i(x)$, which contains the whole state variables. Therefore, the control scheme proposed in this paper can be applied more extensively.

At each step of control design, we utilize the

$$W_i = N_i ||\theta_i||_2^2 = N_i \theta_i^T \theta_i, \quad i = 1, \ldots, n$$

where $\theta_i = [\theta_{i,1}, \ldots, \theta_{i,N_i}]^T$ and N_i are the weight vector and the number of neurons in ith hidden layer, respectively. The operator $||\cdot||_2$ represents the Euclidean norm of column vector. From the definition, we know that W_i is an unknown constant. The adaptive parameter $\hat{W}_i$ is utilized to estimate W_i and the estimated error is $\tilde{W}_i = W_i - \hat{W}_i$. By directly estimating the norm of weight vector θ_i, there are only n adaptive parameters needed to adjust online. Compared with the control algorithms proposed in [36] and [37], the computation in adaptation mechanism utilized in the literature can be reduced from $\sum_{i=1}^n N_i$ to n.

In the following analysis, two control schemes are proposed in the Section III-A and III-B, respectively. The control scheme in Section III-A is for the nonlinear systems with known direction hysteresis, i.e., hysteresis parameter μ_1 is unknown but its sign requires to be known as prior knowledge. In Section III-B, we propose an adaptive neural control algorithm for the nonlinear systems with unknown direction hysteresis.

A. Known Direction Hysteresis

To facilitate the control design, the following assumption is needed.

Assumption 4: Consider the direction of unknown hysteresis is known, i.e., μ_1 is an unknown parameter but its sign must be known. Without loss of generality, it is further assumed $\text{sign}(\mu_1) > 0$.

Theorem 1: Consider the nonstrict-feedback nonlinear system in (1), under the Assumptions 1–4, by selecting the virtual controllers α_j, the auxiliary controller $\bar{v}$ and the actual control signal v as follows:

$$\begin{cases} \alpha_j = -\left(k_j + \frac{n^2}{4j} + 0.5\right)z_j - \frac{1}{2a_j^2} z_j \hat{W}_j - z_{j-1} \\ \bar{v} = -\left(k_n + \frac{n^2}{4n} + 1\right)z_n - \frac{1}{2a_n^2} z_n \hat{W}_n - z_{n-1} \\ v = \hat{e}\bar{v} \end{cases} \tag{16}$$

where $j = 1, \ldots, n-1$ and $z_0 = -\dot{y}_r$. Define a parameter e as $e = 1/\mu_1$, and its estimated value is $\hat{e} = 1/\hat{\mu}_1$. The estimated error is defined as $\tilde{e} = e - \hat{e}$. The adaptive laws are selected as

$$\begin{cases} \dot{\hat{W}}_i = \frac{r_i}{2a_i^2} z_i^2 - k_{0i}\hat{W}_i \qquad (i = 1, \ldots, n) \\ \dot{\hat{e}} = -\eta z_n \bar{v} - \eta_0 \hat{e}. \end{cases} \tag{17}$$

We can construct a robust adaptive neural control scheme to guarantee that all the system states and adaptive parameters are bounded. By increasing the designed parameters $k_1, \ldots, k_n$, $k_{01}, \ldots, k_{0n}, \eta, \eta_0, r_1, \ldots, r_n$ while reducing $a_1, \ldots, a_n$, the tracking error limitation can be adjusted arbitrarily small.

Proof: Backstepping technique is employed to demonstrate this theorem.

Step 1: Differentiating two sides of (13), we have

$$\dot{z}_1 = f_1(x) + h_1(x(t-\tau_1)) + x_2 + \omega_1(t,x) - \dot{y}_r. \tag{18}$$

The Lyapunov–Krasovskii function can be chosen as follows:

$$V_1 = \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{1}{1-\bar{\tau}_{\max}} e^{-\lambda(t-\tau_i)} \int_{t-\tau_i}^{t} e^{\lambda s} z_j^2(s) q_{ij}^2(x(s))ds + \frac{1}{2}z_1^2 + \frac{1}{2r_1}\tilde{W}_1^2. \tag{19}$$

Differentiating V_1 in (19) yields

$$\begin{aligned} \dot{V}_1 = {} & z_1(f_1(x) + h_1(x(t-\tau_1)) + x_2 + \omega_1(t,x) - \dot{y}_r) \\ & - \frac{1}{r_1}\tilde{W}_1\dot{\hat{W}}_1 + \frac{1}{1-\bar{\tau}_{\max}} \sum_{i=1}^{n}\sum_{j=1}^{n} e^{\lambda\tau_i} z_j^2(t) q_{ij}^2(x(t)) \\ & - \frac{(1-\dot{\tau}_i)}{1-\bar{\tau}_{\max}} \sum_{i=1}^{n}\sum_{j=1}^{n} z_j^2(t-\tau_i) q_{ij}^2(x(t-\tau_i)) \\ & - \sum_{i=1}^{n}\sum_{j=1}^{n} \cdot \frac{\lambda(1-\dot{\tau}_i)}{1-\bar{\tau}_{\max}} e^{-\lambda(t-\tau_i)} \int_{t-\tau_i}^{t} e^{\lambda s} z_j^2(s) q_{ij}^2(x(s))ds. \end{aligned} \tag{20}$$

Invoking Assumption 2, one has

$$-\lambda(1-\dot{\tau}_i) \le -\lambda(1-\bar{\tau}_{\max}). \tag{21}$$

By rearranging the terms

$$\sum_{i=1}^{n}\sum_{j=1}^{n} e^{\lambda\tau_i} z_j^2(t) q_{ij}^2(x(t)) = \sum_{j=1}^{n}\sum_{i=1}^{n} e^{\lambda\tau_i} z_j^2(t) q_{ij}^2(x(t))$$

and combining with (20) and (21), we have

$$\begin{aligned} \dot{V}_1 \le {} & z_1(f_1(x) + h_1(x(t-\tau_1)) + z_2 + \alpha_1 + \omega_1(t,x) - \dot{y}_r) \\ & - \frac{1}{r_1}\tilde{W}_1\dot{\hat{W}}_1 + \frac{1}{1-\bar{\tau}_{\max}} \sum_{j=1}^{n}\sum_{i=1}^{n} e^{\lambda\tau_i} z_j^2(t) q_{ij}^2(x(t)) \\ & - \sum_{i=1}^{n}\sum_{j=1}^{n} z_j^2(t-\tau_i) q_{ij}^2(x(t-\tau_i)) - \lambda(1-\bar{\tau}_{\max}) \\ & \times \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{e^{-\lambda(t-\tau_i)}}{1-\bar{\tau}_{\max}} \int_{t-\tau_i}^{t} e^{\lambda s} z_j^2(s) q_{ij}^2(x(s))ds. \end{aligned} \tag{22}$$

Noting that

$$\begin{aligned} z_1 h_1(x(t-\tau_1)) & \le |z_1| \sum_{j=1}^{n} |z_j(t-\tau_1)| q_{1j}(x(t-\tau_1)) \\ & \le \frac{n^2}{4} z_1^2 + \frac{1}{n} \sum_{j=1}^{n} z_j^2(t-\tau_1) q_{1j}^2(x(t-\tau_1)). \end{aligned} \tag{23}$$

Similarly

$$z_1\omega_1(t,x) \le \frac{1}{2}|z_1| + \frac{1}{2}|z_1|\varpi_1^2(x). \tag{24}$$

We define the nonlinear term σ_1 as follows:

$$\sigma_1 = |f_1(x)| + \frac{1}{2} + \frac{1}{2}\varpi_1^2(x) + \frac{\sum_{i=1}^{n} |z_1| e^{\lambda\tau_{\max}} q_{i1}^2(x(t))}{1-\bar{\tau}_{\max}}. \tag{25}$$

With the conclusion of (9), we use the RBFNN $\theta_1^T S_1$ to approximate the nonlinear term σ_1 and the approximated error is δ_1 which satisfies $|\delta_1| \le \varepsilon_1$. Combining with the condition $S_1^T S_1 \le N_1$, we have

$$\begin{aligned} |z_1|\sigma_1 & = |z_1|\left(\theta_1^T S_1 + \delta_1\right) \\ & \le \frac{1}{2a_1^2} z_1^2 ||\theta_1||^2 S_1^T S_1 + \frac{1}{2}a_1^2 + \frac{1}{2}z_1^2 + \frac{1}{2}\varepsilon_1^2 \\ & \le \frac{1}{2a_1^2} z_1^2 W_1 + \frac{1}{2}a_1^2 + \frac{1}{2}z_1^2 + \frac{1}{2}\varepsilon_1^2. \end{aligned} \tag{26}$$

Substituting (23)–(26) into (22), we have

$$\begin{aligned} \dot{V}_1 \le {} & -k_1 z_1^2 + z_1 z_2 + \frac{1}{2}a_1^2 + \frac{1}{2}\varepsilon_1^2 + \frac{\tilde{W}_1}{r_1}\left(\frac{r_1}{2a_1^2}z_1^2 - \dot{\hat{W}}_1\right) \\ & + \frac{1}{1-\bar{\tau}_{\max}} \sum_{j=2}^{n}\sum_{i=1}^{n} e^{\lambda\tau_i} z_j^2(t) q_{ij}^2(x(t)) - \frac{n-1}{n} \\ & \times \sum_{j=1}^{n} z_j^2(t-\tau_1) q_{1j}^2(x(t-\tau_1)) - \lambda(1-\bar{\tau}_{\max}) \\ & \times \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{e^{-\lambda(t-\tau_i)}}{1-\bar{\tau}_{\max}} \int_{t-\tau_i}^{t} e^{\lambda s} z_j^2(s) q_{ij}^2(x(s))ds \\ & - \sum_{i=2}^{n}\sum_{j=1}^{n} z_j^2(t-\tau_i) q_{ij}^2(x(t-\tau_i)). \end{aligned} \tag{27}$$

Step $k(2 \le k \le n-1)$*:* Differentiating two sides of (14) yields

$$\dot{z}_k = f_k(x) + h_k(x(t-\tau_k)) + x_{k+1} + \omega_k(t,x) - \dot{\alpha}_{k-1}. \tag{28}$$

The Lyapunov–Krasovskii function is chosen as

$$V_k = V_{k-1} + \frac{1}{2}z_k^2 + \frac{1}{2r_k}\tilde{W}_k^2. \tag{29}$$

Differentiating V_k in (29), we have

$$\begin{aligned} \dot{V}_k & = \dot{V}_{k-1} + z_k(f_k(x) + h_k(x(t-\tau_k)) + x_{k+1} \\ & \qquad + \omega_k(t,x) - \dot{\alpha}_{k-1}) - \frac{1}{r_k}\tilde{W}_k\dot{\hat{W}}_k \\ & = \dot{V}_{k-1} + z_k(f_k(x) + h_k(x(t-\tau_k)) + z_{k+1} + \alpha_k \\ & \qquad + \omega_k(t,x) - \dot{\alpha}_{k-1}) - \frac{1}{r_k}\tilde{W}_k\dot{\hat{W}}_k. \end{aligned} \tag{30}$$

Since

$$z_k h_k(x(t-\tau_k)) \leq |z_k| \sum_{j=1}^{n} |z_j(t-\tau_k)| q_{kj}(x(t-\tau_k)) \leq \frac{n^2}{4\cdot k} z_k^2 + \frac{k}{n}\sum_{j=1}^{n} z_j^2(t-\tau_k) q_{kj}^2(x(t-\tau_k)). \tag{31}$$

Similarly

$$z_k \omega_k(t,x) \leq \frac{1}{2}|z_k| + \frac{1}{2}|z_k|\varpi_k^2(x). \tag{32}$$

The time differentiation of the virtual control signal α_{k-1} is

$$\dot{\alpha}_{k-1} = \sum_{m=1}^{k-1} \frac{\partial \alpha_{k-1}}{\partial x_m}(f_m(x) + h_m(x(t-\tau_m)) + x_{m+1} + \omega_m(t,x)) + \sum_{m=1}^{k-1}\frac{\partial \alpha_{k-1}}{\hat{W}_m}\dot{\hat{W}}_m + \sum_{m=0}^{k-1}\frac{\partial \alpha_{k-1}}{\partial y_r^{(m)}} y_r^{(m+1)}. \tag{33}$$

By analyzing some complex terms in the $\dot{\alpha}_{k-1}$, we have

$$\begin{aligned} &- z_k \sum_{m=1}^{k-1} \frac{\partial \alpha_{k-1}}{\partial x_m} h_m(x(t-\tau_m)) \\ &\leq |z_k| \sum_{m=1}^{k-1}\left|\frac{\partial \alpha_{k-1}}{\partial x_m}\right| \sum_{j=1}^{n} |z_j(t-\tau_m)| q_{mj}(x(t-\tau_m)) \\ &\leq \sum_{m=1}^{k-1}\left[\frac{n^2}{4} z_k^2 \left(\frac{\partial \alpha_{k-1}}{\partial x_m}\right)^2 + \frac{1}{n}\sum_{j=1}^{n} z_j^2(t-\tau_m) q_{mj}^2(x(t-\tau_m))\right] \\ &\leq \sum_{m=1}^{k-1} \frac{n^2}{4} z_k^2 \left(\frac{\partial \alpha_{k-1}}{\partial x_m}\right)^2 + \frac{1}{n}\sum_{l=1}^{k-1}\sum_{j=1}^{n} z_j^2(t-\tau_l) q_{lj}^2(x(t-\tau_l)). \end{aligned} \tag{34}$$

Noting that

$$-z_k \sum_{m=1}^{k-1} \frac{\partial \alpha_{k-1}}{\partial x_m}\omega_m(t,x) \leq \sum_{m=1}^{k-1}\left[\frac{|z_k|}{2}\left(\frac{\partial \alpha_{k-1}}{\partial x_m}\right)^2 + \frac{|z_k|}{2}\varpi_m^2(x)\right]. \tag{35}$$

We define σ_k as follows:

$$\begin{aligned} \sigma_k =& |f_k(x)| + \frac{1}{2} + \sum_{m=1}^{k}\frac{1}{2}\varpi_m^2(x) + \sum_{m=1}^{k-1}\left|\frac{\partial \alpha_{k-1}}{\partial x_m}\right| |f_m(x)| \\ &+ \sum_{m=1}^{k-1}\left(\frac{n^2}{4}|z_k| + \frac{1}{2}\right)\left(\frac{\partial \alpha_{k-1}}{\partial x_m}\right)^2 + \sum_{m=1}^{k-1}\left|\frac{\partial \alpha_{k-1}}{\partial x_m}\right| |x_{m+1}| \\ &+ \sum_{m=1}^{k-1}\frac{\partial \alpha_{k-1}}{\hat{W}_m}\dot{\hat{W}}_m + \sum_{m=0}^{k-1}\frac{\partial \alpha_{k-1}}{\partial y_r^{(m)}} y_r^{(m+1)} \\ &+ \frac{1}{1-\bar{\tau}_{\max}}\sum_{i=1}^{n} e^{\lambda \tau_{\max}} |z_k| q_{ik}^2(x(t)). \end{aligned} \tag{36}$$

We employ the RBFNN to approximate the nonlinear term σ_k, the approximated error is δ_k which satisfies $|\delta_k| \leq \varepsilon_k$. Combining with the condition $S_k^T S_k \leq N_k$, we have

$$|z_k|\sigma_k = |z_k|\left(\theta_k^T S_k + \delta_k\right) \leq \frac{1}{2a_k^2} z_k^2 W_k + \frac{1}{2}a_k^2 + \frac{1}{2}z_k^2 + \frac{1}{2}\varepsilon_k^2. \tag{37}$$

Substituting (31)–(37) into (30), we have

$$\begin{aligned} \dot{V}_k \leq& -\sum_{j=1}^{k} k_j z_j^2 + z_k z_{k+1} + \frac{1}{2}\sum_{j=1}^{k}\left(a_j^2 + \varepsilon_j^2\right) \\ &+ \sum_{j=1}^{k}\frac{\tilde{W}_j}{r_j}\left(\frac{r_j}{2a_j^2} z_j^2 - \dot{\hat{W}}_j\right) + \frac{\sum_{j=k+1}^{n}\sum_{i=1}^{n} e^{\lambda \tau_i} z_j^2(t) q_{ij}^2(x(t))}{1-\bar{\tau}_{\max}} \\ &- \sum_{i=k+1}^{n}\sum_{j=1}^{n} z_j^2(t-\tau_i) q_{ij}^2(x(t-\tau_i)) - \frac{n-k}{n} \\ &\cdot \sum_{l=1}^{k}\sum_{j=1}^{n} z_j^2(t-\tau_l) q_{lj}^2(x(t-\tau_l)) - \lambda(1-\bar{\tau}_{\max}) \\ &\cdot \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{e^{-\lambda(t-\tau_i)}}{1-\bar{\tau}_{\max}}\int_{t-\tau_i}^{t} e^{\lambda s} z_j^2(s) q_{ij}^2(x(s)) ds. \end{aligned} \tag{38}$$

Step n: The Lyapunov–Krasovskii function can be chosen as follows:

$$V_n = V_{n-1} + \frac{1}{2}z_n^2 + \frac{1}{2r_n}\tilde{W}_n^2 + \frac{\mu_1}{2\eta}\tilde{e}^2. \tag{39}$$

Differentiating two sides of (39), we have

$$\dot{V}_n = \dot{V}_{n-1} + z_n(f_n(x) + h_n(x(t-\tau_n)) + H(v) + \omega_n(t,x) - \dot{\alpha}_{n-1}) - \frac{1}{r_n}\tilde{W}_n\dot{\hat{W}}_n - \frac{\mu_1}{\eta}\tilde{e}\dot{\hat{e}}. \tag{40}$$

Since

$$z_n h_n(x(t-\tau_n)) \leq \frac{n^2}{4\cdot n} z_n^2 + \frac{n}{n}\sum_{j=1}^{n} z_j^2(t-\tau_k) q_{kj}^2(x(t-\tau_k)). \tag{41}$$

Noting that

$$z_n \omega_n(t,x) \leq \frac{1}{2}|z_n| + \frac{1}{2}|z_n|\varpi_n^2(x). \tag{42}$$

The time differentiation of the virtual controller α_{n-1} is

$$\dot{\alpha}_{n-1} = \sum_{m=1}^{n-1}\frac{\partial \alpha_{n-1}}{\partial x_m}(f_m(x) + h_m(x(t-\tau_m)) + x_{m+1} + \omega_m(t,x)) + \sum_{m=1}^{n-1}\frac{\partial \alpha_{n-1}}{\hat{W}_m}\dot{\hat{W}}_m + \sum_{m=0}^{n-1}\frac{\partial \alpha_{n-1}}{\partial y_r^{(m)}} y_r^{(m+1)}. \tag{43}$$

Noting that

$$-z_n \sum_{m=1}^{n-1} \frac{\partial \alpha_{k-1}}{\partial x_m} h_m(x(t-\tau_m)) \leq \sum_{m=1}^{n-1} \frac{n^2}{4} z_k^2 \left(\frac{\partial \alpha_{k-1}}{\partial x_m}\right)^2 + \frac{1}{n}\sum_{l=1}^{n-1}\sum_{j=1}^{n} z_j^2(t-\tau_l) q_{lj}^2(x(t-\tau_l)). \quad (44)$$

Subsequently, a compounded nonlinear term σ_n similar to σ_k in the equality (36) is also defined. By using the RBFNN to approximate the unknown nonlinear term σ_n, and the approximated error is δ_n satisfying $|\delta_n| \leq \varepsilon_n$. Thus, we have

$$|z_n|\sigma_n = |z_n|\left(\theta_n^T S_n + \delta_n\right) \leq \frac{1}{2a_n^2} z_n^2 W_n + \frac{1}{2}a_n^2 + \frac{1}{2}z_n^2 + \frac{1}{2}\varepsilon_n^2. \quad (45)$$

The hysteresis input $H(\upsilon)$ in (40) can be dealt with as follows:

$$z_n H(\upsilon) = z_n(\mu_1 \upsilon + \mu_2 \zeta). \quad (46)$$

Consider μ_1 is the unknown constant, we should utilize $\hat{\mu}_1$ to estimate μ_1. By defining $e = 1/\mu_1$ and $\hat{e} = 1/\hat{\mu}_1$, thus, the estimated error is $\tilde{e} = e - \hat{e}$. Combining with Assumption 4 yields

$$\begin{aligned} z_n H(\upsilon) &= z_n(\mu_1 \upsilon + \mu_2 \zeta) = z_n \mu_1 \hat{e} \bar{\upsilon} + z_n \mu_2 \zeta \\ &= z_n \mu_1 \left(\frac{1}{\mu_1} - \frac{1}{\tilde{\mu}_1}\right)\bar{\upsilon} + z_n \mu_2 \zeta \\ &= z_n \bar{\upsilon} - z_n \mu_1 \tilde{e} \bar{\upsilon} + z_n \mu_2 \zeta. \end{aligned} \quad (47)$$

Since

$$z_n \mu_2 \zeta \leq \frac{1}{2}z_n^2 + \frac{1}{2}(\mu_2 \bar{\zeta})^2 \quad (48)$$

where $\bar{\zeta} = \sqrt[n]{1/(\beta + \chi)}$.

From (16), the auxiliary control signal $\bar{\upsilon}$ is designed as

$$\bar{\upsilon} = -\left(k_n + \frac{n^2}{4 \cdot n} + 0.5 + 0.5\right) z_n - \frac{1}{2a_n^2} z_n \hat{W}_n - z_{n-1}. \quad (49)$$

Based on the analysis of (40)–(49), we have

$$\begin{aligned} \dot{V}_n \leq &- \sum_{j=1}^{n} k_j z_j^2 + \frac{1}{2}\sum_{j=1}^{n}\left(a_j^2 + \varepsilon_j^2\right) + \sum_{j=1}^{n} \frac{\tilde{W}_j}{r_j}\left(\frac{r_j}{2a_j^2} z_j^2 - \dot{\hat{W}}_j\right) \\ &- \lambda \sum_{i=1}^{n}\sum_{j=1}^{n} e^{-\lambda(t-\tau_i)} \int_{t-\tau_i}^{t} e^{\lambda s} z_j^2(s) q_{ij}^2(x(s)) ds \\ &- \mu_1 \tilde{e}\left(z_n \bar{\upsilon} + \frac{\dot{\hat{e}}}{\eta}\right) + \frac{1}{2}(\mu_2 \bar{\zeta})^2. \end{aligned} \quad (50)$$

Noting that

$$\sum_{j=1}^{n} \frac{k_{0j}}{r_j} \tilde{W}_j \hat{W}_j \leq -\sum_{j=1}^{n} \frac{k_{0j}}{2r_j} \tilde{W}_j^2 + \sum_{j=1}^{n} \frac{k_{0j}}{2r_j} W_j^2 \quad (51)$$

$$\frac{\mu_1 \eta_0}{\eta} \tilde{e}\hat{e} \leq -\frac{\mu_1 \eta_0}{2\eta}\tilde{e}^2 + \frac{\mu_1 \eta_0}{2\eta} e^2. \quad (52)$$

Substituting (51) and (52) into (50), we have

$$\begin{aligned} \dot{V}_n \leq &- \sum_{j=1}^{n} k_j z_j^2 - \sum_{j=1}^{n} \frac{k_{0j}}{2r_j}\tilde{W}_j^2 + \sum_{j=1}^{n} \frac{k_{0j}}{2r_j} W_j^2 + \frac{1}{2}\sum_{j=1}^{n}\left(a_j^2 + \varepsilon_j^2\right) \\ &- \lambda \sum_{i=1}^{n}\sum_{j=1}^{n} e^{-\lambda(t-\tau_i)} \int_{t-\tau_i}^{t} e^{\lambda s} z_j^2(s) q_{ij}^2(x(s)) ds \\ &+ \frac{1}{2}(\mu_2 \bar{\zeta})^2 - \frac{\mu_1 \eta_0}{2\eta}\tilde{e}^2 + \frac{\mu_1 \eta_0}{2\eta} e^2. \end{aligned} \quad (53)$$

Let $\mathcal{P} = \min\{2k_1, \ldots, 2k_n, k_{01}, \ldots, k_{0n}, \lambda(1 - \bar{\tau}_{\max}), \eta_0\}$ and

$$\mathcal{Y} = \sum_{j=1}^{n} k_{0j}/2r_j W_j^2 + 1/2 \sum_{j=1}^{n}\left(a_j^2 + \varepsilon_j^2\right) + \mu_1 \eta_0 / 2\eta e^2 + 1/2(\mu_2 \bar{\zeta})^2.$$

Consequently, the inequality (53) can be reexpressed as

$$\dot{V}_n \leq -\mathcal{P} V_n + \mathcal{Y}. \quad (54)$$

Thus, we have

$$V_n \leq \left(V_n(0) - \frac{\mathcal{Y}}{\mathcal{P}}\right) e^{-\mathcal{P}t} + \frac{\mathcal{Y}}{\mathcal{P}}. \quad (55)$$

Noting that if $t \to \infty$

$$z_1^2 = (y - y_r)^2 \leq \frac{2\mathcal{Y}}{\mathcal{P}}. \quad (56)$$

By increasing the parameters k_j, k_{0j}, η, and r_j, η_0 meanwhile reducing the parameters a_j, the tracking error limitation $2\mathcal{Y}/\mathcal{P}$ can be adjusted arbitrarily small. Therefore, the control scheme proposed in Theorem 1 can guarantee that all the signals in the closed-loop system remain bounded, and the output of the system converges to an adjustable neighborhood of the reference signal. ■

Remark 6: With the prerequisite of Assumption 4, a robust adaptive neural controller can be constructed for the nonstrict-feedback system in (1), and the conclusion is presented in Theorem 1. However, from Assumption 4, we know that the sign of μ_1 must be known in advance as prior knowledge. To relax the restriction, a Nussbaum-type function is introduced to solve the problem of unknown control direction and a novel adaptive neural control scheme is proposed in the following theorem.

B. Unknown Direction Hysteresis

In the section, we investigate the fusion of unknown direction hysteresis with adaptive neural control approach. The stable control algorithm is presented in the following theorem.

Theorem 2: Consider the nonstrict-feedback system in (1), under the Assumptions 1–3, if we choose the virtual controllers α_j, the auxiliary controller $\bar{\upsilon}$, the control law υ, and the adaptive laws as follows:

$$\begin{cases} \alpha_j = -(k_j + \frac{n^2}{4j} + 0.5) z_j - \frac{1}{2a_j^2} z_j \hat{W}_j - z_{j-1} \\ \bar{\upsilon} = -(k_n + \frac{n^2}{4n} + 1) z_n - \frac{1}{2a_n^2} z_n \hat{W}_n - z_{n-1} \\ \upsilon = -N(\vartheta)\bar{\upsilon} \\ \dot{\vartheta} = -\gamma\, \bar{\upsilon} z_n \\ \dot{\hat{W}}_i = \frac{r_i}{2a_i^2} z_i^2 - k_{0i}\hat{W}_i \qquad (i = 1, \ldots, n; \quad j = 1, \ldots, n-1) \end{cases} \quad (57)$$

where $z_0 = -\dot{y}_r$. We can construct a robust adaptive neural control scheme to guarantee that all system states and adaptive parameters are bounded. By increasing the designed parameters $k_1, \ldots, k_n, k_{01}, \ldots, k_{0n}, \gamma, r_1, \ldots, r_n$ while reducing $a_1, \ldots, a_n$, the tracking error limitation can be adjusted arbitrarily small.

Remark 7: It should be highlighted that Assumption 4 is not necessary to the control scheme in Theorem 2. Therefore, the control algorithm in Theorem 2 can be applied to nonstrict-feedback system with the unknown direction hysteresis.

Proof: We employ the backstepping technique to demonstrate this theorem. The first $n-1$ steps are similar to the aforementioned procedures. At nth step, the Lyapunov–Krasovskii function is chosen as follows:

$$V_n = V_{n-1} + \frac{1}{2}z_n^2 + \frac{1}{2r_n}\tilde{W}_n^2. \tag{58}$$

Differentiating two sides of (58), we have

$$\dot{V}_n = \dot{V}_{n-1} + z_n(f_n(x) + h_n(x(t-\tau_n)) + H(\upsilon) + \omega_n(t,x) - \dot{\alpha}_{n-1}) - \frac{1}{r_n}\tilde{W}_n\dot{\hat{W}}_n. \tag{59}$$

Noting that

$$z_n H(\upsilon) = z_n(\mu_1\upsilon + \mu_2\zeta). \tag{60}$$

Consider the sign of μ_1 is unknown, we introduce a Nussbaum-type function $N(\vartheta)$ as $N(\vartheta) = \vartheta^2\cos\vartheta$. The control signal υ is constructed as $\upsilon = -N(\vartheta)\bar{\upsilon}$. Subsequently, we have

$$z_n H(\upsilon) = -(\mu_1 N(\vartheta) + 1)z_n\bar{\upsilon} + z_n\bar{\upsilon} + z_n\mu_2\zeta \tag{61}$$

where $\bar{\upsilon}$ is the auxiliary control signal needed to be further designed. The variable ϑ is obtained by the differentiation equation $\dot{\vartheta} = -\gamma\bar{\upsilon}z_n$. With these analysis, equality (61) can be expressed as

$$z_n H(\upsilon) = \frac{1}{\gamma}(\mu_1 N(\vartheta) + 1)\dot{\vartheta} + z_n\bar{\upsilon} + z_n\mu_2\zeta. \tag{62}$$

The auxiliary controller $\bar{\upsilon}$ and the adaptive parameters $\hat{W}_i$ are designed in (57), respectively. By repeating the procedures of (18)–(38), and combining with the analysis of (59)–(62), we have

$$\begin{aligned}\dot{V}_n \le & -\sum_{j=1}^{n} k_j z_j^2 - \sum_{j=1}^{n}\frac{k_{0j}}{2r_j}\tilde{W}_j^2 + \sum_{j=1}^{n}\frac{k_{0j}}{2r_j}W_j^2 + \frac{1}{2}\sum_{j=1}^{n}\left(a_j^2 + \varepsilon_j^2\right)\\ & -\lambda\sum_{i=1}^{n}\sum_{j=1}^{n} e^{-\lambda(t-\tau_i)}\int_{t-\tau_i}^{t} e^{\lambda s} z_j^2(s) q_{ij}^2(x(s))ds\\ & + \frac{1}{\gamma}(\mu_1 N(\vartheta) + 1)\dot{\vartheta} + \frac{1}{2}(\mu_2\bar{\zeta}).^2\end{aligned} \tag{63}$$

Two constants are defined as $\mathcal{P} = \min\{2k_1, \ldots, 2k_n, k_{01}, \ldots, k_{0n}, \lambda(1-\bar{\tau}_{\max})\}$ and

$$\mathcal{Y} = \sum_{j=1}^{n} k_{0j}/2r_j W_j^2 + 1/2\sum_{j=1}^{n}\left(a_j^2 + \varepsilon_j^2\right) + 1/2(\mu_2\bar{\zeta})^2.$$

Subsequently, inequality (63) can be written as the following form:

$$\dot{V}_n \le -\mathcal{P}V_n + \mathcal{Y} + \frac{1}{\gamma}(\mu_1 N(\vartheta) + 1)\dot{\vartheta}. \tag{64}$$

By direct integration of the differentiation inequality (64) at the interval $[0, t)$ yields

$$V_n \le V_n(0)e^{-\mathcal{P}t} + \frac{\mathcal{Y}}{\mathcal{P}}(1 - e^{-\mathcal{P}t}) + \frac{e^{-\mathcal{P}t}}{\gamma}\int_0^t (\mu_1 N(\vartheta) + 1)\dot{\vartheta}\cdot e^{\mathcal{P}\tau}d\tau. \tag{65}$$

From Lemma 2, we can know that the terms V_n, ϑ, and $\int_0^t(\mu_1 N(\vartheta) + 1)\dot{\vartheta}d\tau$ are bounded at the interval $[0, t)$. Furthermore, the result can be extended to $t \to \infty$. Consequently, let $\mathcal{C}_0$ be upper bound of $\Theta = \int_0^t(\mu_1 N(\vartheta) + 1)\dot{\vartheta}e^{-\mathcal{P}(t-\tau)}d\tau$. Then, the following inequality holds:

$$V_n \le \left(V_n(0) - \frac{\mathcal{Y}}{\mathcal{P}}\right)e^{-\mathcal{P}t} + \frac{\mathcal{Y}}{\mathcal{P}} + \frac{\mathcal{C}_0}{\gamma}. \tag{66}$$

When $t \to \infty$

$$z_1^2 = (y - y_r)^2 \le \frac{2\mathcal{Y}}{\mathcal{P}} + \frac{2\mathcal{C}_0}{\gamma}. \tag{67}$$

By increasing the designed parameters k_j, λ, γ, and k_{0j} meanwhile reducing the parameters a_j, the tracking error limitation can be adjusted arbitrarily small. Therefore, the restriction in Assumption 4 can be removed by the control scheme presented in the Theorem 2. Furthermore, the control algorithm proposed in the Theorem 2 can guarantee that all the state variables and adaptive parameters of the control system are bounded. ■

IV. Simulation Study

In the section, three numerical examples are provided to validate the effectiveness of the proposed control schemes. Example 1 is a general nonlinear system model with the time-varying delays and the unknown direction hysteresis. Examples 2 and 3 are practical physical models.

Example 1: Consider the following general third-order nonstrict-feedback nonlinear system:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0\end{bmatrix}\cdot\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}f_1(x)\\ f_2(x)\\ f_3(x)\end{bmatrix} + \begin{bmatrix}h_1(\tau_1(t))\\ h_2(\tau_2(t))\\ h_3(\tau_3(t))\end{bmatrix} + \begin{bmatrix}0.1\sin(t)x_1x_2^2x_3\\ 0.3\cos(\pi t)x_1^2x_2^2\\ 0.1\sin(\pi t)x_1x_2^2\end{bmatrix} + \begin{bmatrix}0\\ 0\\ u\end{bmatrix},\quad u = H(\upsilon) \tag{68}$$

where $y = x_1$ is the output of control system. The unknown nonlinear functions are selected as $f_1(x) = x_1^2x_2 + x_3$, $f_2(x) = x_1^2x_2 + 2x_3$, and $f_3(x) = -7x_1 - 10x_2^2 - 5x_2$, respectively. It should be noted that the external disturbances $\omega_1 = 0.1\sin(t)x_1x_2^2x_3$, $\omega_2 = 0.3\cos(\pi t)x_1^2x_2^2$, and $\omega_3 = 0.1\sin(\pi t)x_1x_2^2$ contain whole state variables and time variable simultaneously. The time-delayed terms are selected as $h_1(\tau_1(t)) = x_2(t-\tau_1(t))x_3(t-\tau_1(t))$, $h_2(\tau_2(t)) = 0.5x_2^2(t-\tau_2(t))x_3(t-\tau_2(t))$, and $h_3(\tau_3(t)) = 0.01x_1^2(t-\tau_3(t))x_3^2$

TABLE II
INITIAL VALUES AND DESIGNED PARAMETERS

$x_1(0)$	$x_2(0)$	$x_3(0)$	$\hat{W}_1(0)$	$\hat{W}_2(0)$	$\hat{W}_3(0)$	$\hat{e}(0)$
0.3	0.4	0	0.5	0.6	0.7	0
k_1	k_2	k_3	a_1	a_2	a_3	η_0
10	11	12	2	1.8	1.85	0.5
k_{01}	k_{02}	k_{03}	r_1	r_2	r_3	η
0.0008	0.0025	0.0002	200	350	300	1

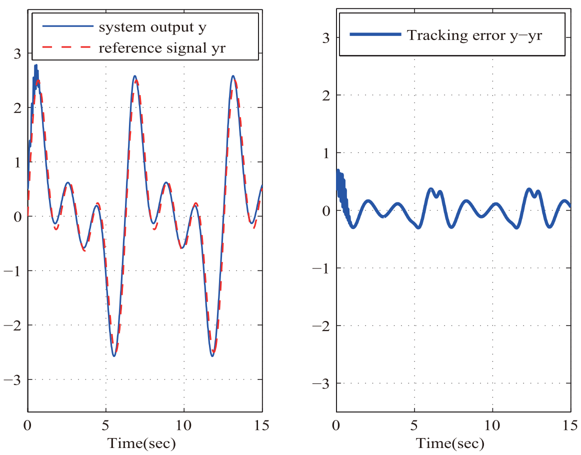

Fig. 3. Trajectories of system output y and the corresponding tracking error $z_1 = y - y_r$.

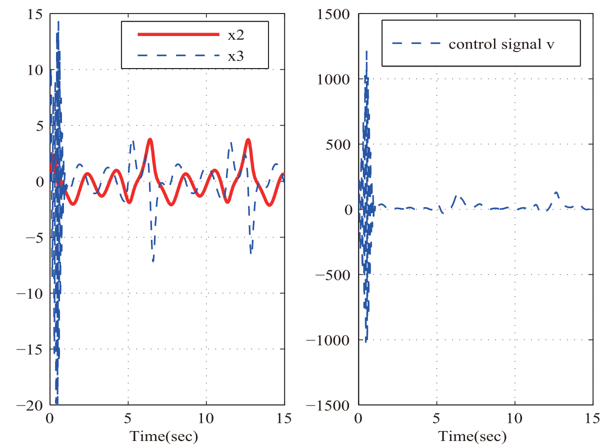

Fig. 4. Trajectories of system states x_2, x_3 and actual control signal v.

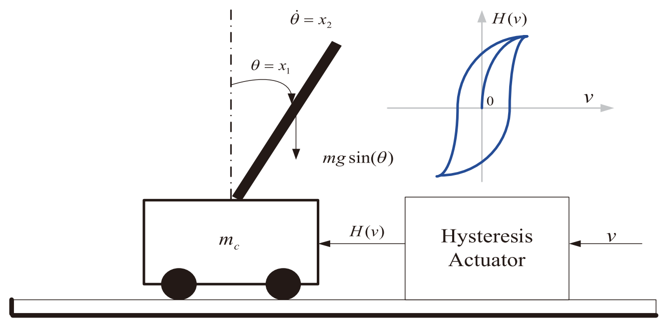

Fig. 5. Inverted pendulum system with hysteresis actuator.

$(t - \tau_3(t))$, respectively. The time delays are selected as $\tau_1(t) = 0.5 + 0.1\sin(t)$, $\tau_2(t) = 0.5 + 0.3\sin(0.5t)$, and $\tau_3(t) = 0.5 + 0.2\sin(\pi t)$, respectively.

The actual control signal is v, and the reference signal is $y_r = \sin(t) + \sin(2t) + \cos(3t)$. The control objective of this simulation is to construct a robust adaptive neural control scheme to guarantee that all the system states and adaptive parameters are bounded, and the tracking error can converge to an adjustable neighborhood of the origin.

In the simulation, the hysteresis parameters are presented in Table I, and the initial values of system states and adaptive parameters are provided in Table II. In addition, the designed parameters are also given in Table II. Based on the theorem derived in this paper and the aforementioned designed parameters, a robust adaptive neural control scheme is constructed. The simulation results are shown in Figs. 3 and 4, respectively. Fig. 3 shows the tracking performance of control system. Fig. 4 shows that the trajectories of system states x_2, x_3 and the corresponding actual control signal v.

Example 2: Consider the following inverted pendulum with hysteresis actuator shown in Fig. 5. By writing $x_1 = \theta$ and $x_2 = \dot{\theta}$, the inverted pendulum can be modeled as the following second-order differentiation equation [45]:

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= f(\bar{x}_2) + g(\bar{x}_2)H(v) + d(t)\end{aligned} \tag{69}$$

where

$$f(\bar{x}_2) = \frac{9.8(m_c+m)\sin x_1 - mlx_2^2\cos x_1 \sin x_1}{l\left(\frac{4}{3} - \frac{m\cos^2 x_1}{m_c+m}\right)(m_c+m)} \tag{70}$$

$$g(\bar{x}_2) = \frac{\cos x_1}{l\left(\frac{4}{3} - \frac{m\cos^2 x_1}{m_c+m}\right)(m_c+m)} \tag{71}$$

$$d(t) = 3 + 2\cos(2t). \tag{72}$$

The physical parameters are selected as $m_c = 1$ kg, $m = 0.1$ kg, and $l = 0.5$ m, respectively.

It should be noted that the simulation model has been performed in [45]. However, the works in [45] considered the known direction backlash-like hysteresis model. Compared with the simulation model in [45], we consider a more general modified Bouc–Wen hysteresis model in this paper. Furthermore, the hysteresis direction does not require to be known. By constructing the stable control scheme as the Theorem 2, the simulation results are shown in Figs. 6 and 7, respectively. The comparative simulation results can further validate the effectiveness of the proposed control scheme in this paper.

Example 3: Consider the following piezo-positioning mechanism subjected to unknown direction hysteresis input:

$$M\ddot{y}(t) + D\dot{y}(t) + Fy(t) = u, \quad u = H(v) \tag{73}$$

where y, $\dot{y}$, and $\ddot{y}$ represent the position, the velocity, and the acceleration, respectively. v is the voltage signal applied to the piezo-positioning mechanic system. M, D, and F denote

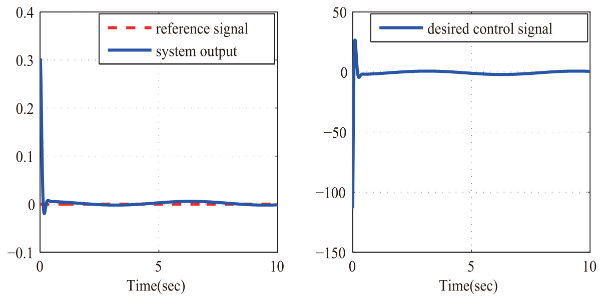

Fig. 6. Trajectories of the system output y and control signal υ.

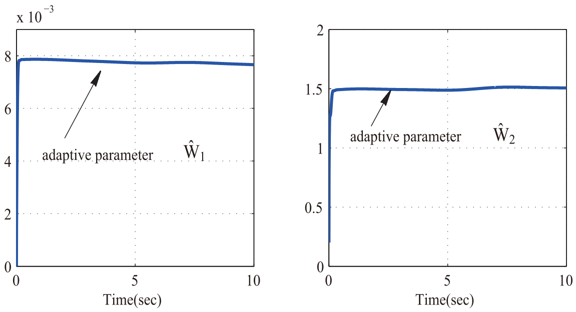

Fig. 7. Trajectories of adaptive parameters $\hat{W}_1$ and $\hat{W}_2$.

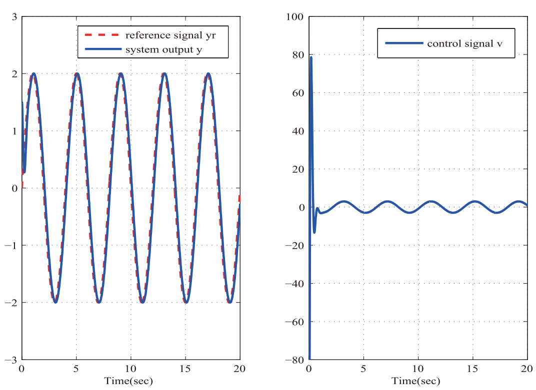

Fig. 8. Trajectories of the system output y and control signal υ.

the mass, damping, and stiffness coefficients, respectively. These parameters are all unknown. In the simulation, the actual parameter values are chosen as $M = 1$ kg, $D = 0.15$ Ns/m, $F = 1$ M/m. The modified Bouc–Wen hysteresis parameters are $\mu_1 = \mu_2 = 1$, $\beta = 1$, $\chi = 0.5$, and $n = 2$. The initial value of state is chosen as $y(0) = 1.5$. The reference signal is $y_r = 2\sin(0.5\pi t)$.

Remark 8: The piezo-position mechanic model in (73) has been investigated in [41] and [75]. However, the control schemes in [41] and [75] require both the parameters and the direction of modified Bouc–Wen hysteresis to be known as prior knowledge. To relax these restrictions, an adaptive control algorithm is proposed in Theorem 2 for the nonlinear systems with the unknown direction hysteresis input.

In the simulation, the initial values of adaptive parameters are chosen as $\hat{W}_1(0) = 0.1$, $\hat{W}_2(0) = 0.01$, and $\vartheta(0) = 0.1$, respectively. The designed parameters are selected as $k_1 = 15$, $k_2 = 16$, $a_1 = a_2 = 10$, $r_1 = 250$, $r_2 = 250$, $\gamma = 1$, $k_{01} = 0.003$, and $k_{02} = 0.002$, respectively. Therefore, we can construct a neural-network-based adaptive controller as Theorem 2. The simulation results are shown in Figs. 8 and 9, respectively. Fig. 8 shows that the tracking trajectory and the corresponding actual control signal υ. Fig. 9 shows the trajectories of adaptive parameters $\hat{W}_1$ and $\hat{W}_2$. Clearly, the simulation results can further validate the effectiveness of the proposed control scheme in this paper.

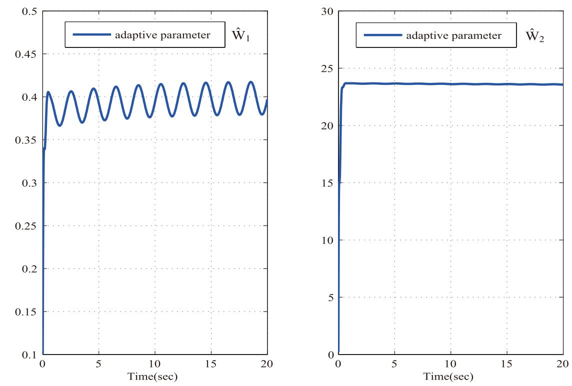

Fig. 9. Trajectories of adaptive parameters $\hat{W}_1$ and $\hat{W}_2$.

V. Conclusion

In this paper, two NN-based adaptive control algorithms are proposed for a class of CT nonstrict-feedback systems with unknown modified Bouc–Wen hysteresis input and time varying delays. Compared with the previous works on hysteresis phenomenon, the direction of the modified Bouc–Wen hysteresis model studied in the literature can be unknown. Furthermore, an optimized method is proposed to reduce the computation burden in adaptation mechanism. Based on the Lyapunov–Krasovskii method, two adaptive neural control schemes are constructed to guarantee that all the system states and adaptive parameters remain bounded, and the tracking error is driven to an adjustable neighborhood of the origin. In addition, three simulation examples are provided to verify the effectiveness of the proposed control schemes.

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